From Conjectures to Formal Proofs

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Related K-5 Standards and "Illustrations"

Gr	Domain	Cluster/Standards and Illustrations
K	Geometry	**Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).** • **K.G Shape Hunt Part 1** • **K.G Shape Hunt Part 2** • **K.G Shape Sequence Search**
K	Geometry	2. Correctly name shapes regardless of their orientations or overall size.
K	Geometry	**Analyze, compare, create, and compose shapes.**
K	Geometry	4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length). • **K.G Alike or Different Game**
1	Geometry	**Reason with shapes and their attributes.**
1	Geometry	1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes. • **1.G 3-D Shape Sort** • **1.G All vs. Only some**
2	Geometry	**Reason with shapes and their attributes.**
2	Geometry	1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. • **2.G Polygons**
3	Geometry	**Reason with shapes and their attributes.**
3	Geometry	1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
4	Geometry	**Draw and identify lines and angles, and classify shapes by properties of their lines and angles.**
4	Geometry	1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. • **4.G The Geometry of Letters** • **4.G What's the Point?** • **4.MD,G Measuring Angles**
4	Geometry	2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. • **4.G Are these right?** • **4.G Defining Attributes of Rectangles and Parallelograms** • **4.G What is a Trapezoid? (Part 1)** • **4.G What shape am I?** • **4.MD,G Finding an unknown angle**

Gr	Domain	Cluster/Standards and Illustrations
4	Geometry	3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry. • **4.G Finding Lines of Symmetry** • **4.G Lines of symmetry for circles** • **4.G Lines of symmetry for quadrilaterals** • **4.G Lines of symmetry for triangles**
4	Measurement and Data	**Geometric measurement: understand concepts of angle and measure angles.**
4	Measurement and Data	**5**. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: **5.a**. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. **5.b**. An angle that turns through **n** one-degree angles is said to have an angle measure of **n** degrees.
4	Measurement and Data	**7.** Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. • **4.MD,G Finding an unknown angle** • **4.MD,G Measuring Angles**
5	Geometry	**Graph points on the coordinate plane to solve real-world and mathematical problems.**
5	Geometry	**1.** Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g.,**x**-axis and **x**-coordinate, **y**-axis and **y**-coordinate). • **5.G Battle Ship Using Grid Paper**
5	Geometry	2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. • **5.G Meerkat Coordinate Plane Task**
5	Geometry	**Classify two-dimensional figures into categories based on their properties.**
5	Geometry	3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles. • **5.G Always, Sometimes, Never**
5	Geometry	4. Classify two-dimensional figures in a hierarchy based on properties. • **5.G What is a Trapezoid? (Part 2)**

Related 6-8 Standards and "Illustrations"

Gr	Domain	**Cluster**/Standards and **Illustrations**
6	Geometry	**Solve real-world and mathematical problems involving area, surface area, and volume.**
6	Geometry	3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. • **6.G Polygons in the Coordinate Plane** • **6.G Walking the Block**
7	Geometry	**Draw, construct, and describe geometrical figures and describe the relationships between them.**
7	Geometry	1. Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. • **7.G, 7.RP, 8.G Scaling angles and polygons** • **7.G Approximating the area of a circle** • **7.G Circumference of a Circle** • **7.G Floor Plan** • **7.G Map distance** • **7.G Rescaling Washington Park**
7	Geometry	2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
7	Geometry	**Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**
7	Geometry	5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and use them to solve simple equations for an unknown angle in a figure.
8	Geometry	**Understand congruence and similarity using physical models, transparencies, or geometry software.** • **7.G, 7.RP, 8.G Scaling angles and polygons** • **8.G A scaled curve** • **8.G, G-GPE, G-SRT, G-CO Is this a rectangle?** • **8.G Partitioning a hexagon** • **8.G Reflecting a rectangle over a diagonal** • **8.G Same Size, Same Shape?**
8	Geometry	1. Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.
8	Geometry	2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. • **8.G Circle Sandwich** • **8.G Congruent Rectangles** • **8.G Congruent Segments** • **8.G Congruent Triangles** • **8.G Cutting a rectangle into two congruent triangles** • **8.G Triangle congruence with coordinates**
8	Geometry	3. Describe the effects of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. • **8.G.A.3 Effects of Dilations on Length, Area, and Angles** • **8.G Point Reflection** • **8.G Reflecting reflections** • **8.G Triangle congruence with coordinates**

Gr	Domain	Cluster/Standards and Illustrations
8	Geometry	4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. • **8.G, 8.EE Different Areas?** • **8.G.A.4 Are They Similar?** • **8.G Creating Similar Triangles**
8	Geometry	5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so. • **8.G.A.5 Street Intersections** • **8.G A Triangle's Interior Angles** • **8.G Congruence of Alternate Interior Angles via Rotations** • **8.G Find the Angle** • **8.G Find the Missing Angle** • **8.G Rigid motions and congruent angles** • **8.G Tile Patterns II: hexagons** • **8.G Tile Patterns I: octagons and squares**
8	Geometry	**Understand and apply the Pythagorean Theorem.** • **8.G Applying the Pythagorean Theorem in a mathematical context** • **8.G A rectangle in the coordinate plane** • **8.G Bird and Dog Race** • **8.G.B Sizing up Squares** • **8.G, G-GPE, G-SRT, G-CO Is this a rectangle?**
8	Geometry	6. **Explain** a proof of the Pythagorean Theorem and its converse. • **8.G Converse of the Pythagorean Theorem**
8	Geometry	8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. • **8.G Finding isosceles triangles** • **8.G Finding the distance between points**

Related HS Standards and "Illustrations"

Domain--Congruence	
Illustrations	**Cluster/Standard**
• **G-C, G-CO Tangent Lines and the Radius of a Circle** • **G-CO.6 Symmetries of a circle** • **G-GPE, G-CO, G-SRT Unit Squares and Triangles**	**G.CO.A** ***Experiment with transformations in the plane***
• **G-CO Defining Parallel Lines** • **G-CO Defining Perpendicular Lines**	**1.** Know precise **definitions** of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
• **G-CO Dilations and Distances** • **G-CO Fixed points of rigid motions** • **G-CO Horizontal Stretch of the Plane**	**2.** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
• **G-CO Origami regular octagon** • **G-CO Seven Circles II** • **G-CO Symmetries of a quadrilateral I** • **G-CO Symmetries of a quadrilateral II** • **G-CO Symmetries of rectangles**	**3.** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
• **G-CO Defining Reflections** • **G-CO Defining Rotations** • **G-CO Identifying Rotations** • **G-CO Identifying Translations**	**4.** Develop **definitions** of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

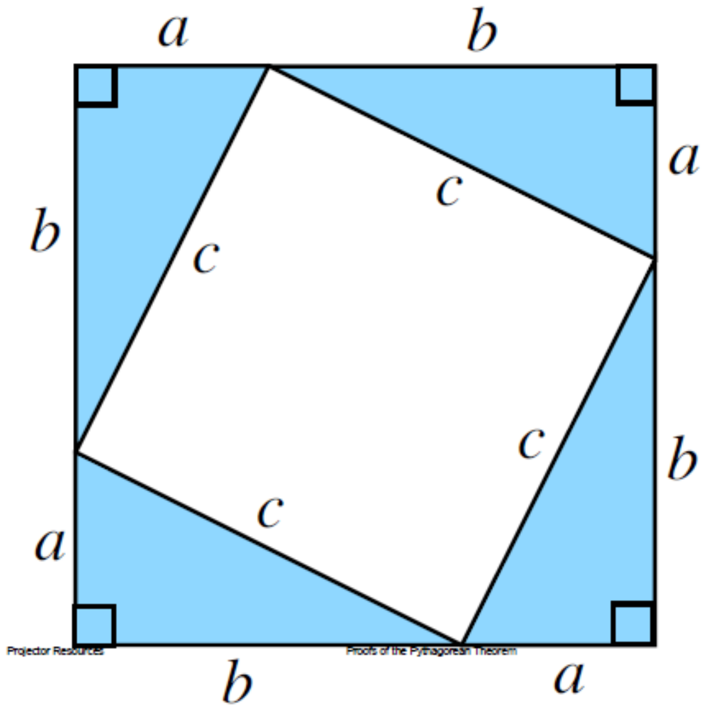
Illustrations	Cluster/Standard
• **F-TF, G-CO, Trigonometric Identities and Rigid Motions** • **G-CO Reflected Triangles** • **G-CO Showing a triangle congruence: a particular case** • **G-CO Showing a triangle congruence: the general case**	**5.** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
• **8.G, G-GPE, G-SRT, G-CO Is this a rectangle?** • **G-CO Are the Triangles Congruent?** • **G-CO Reflections and Equilateral Triangles** • **G.CO Reflections and Equilateral Triangles II** • **G.CO Reflections and Isosceles Triangles**	**G.CO.B** ***Understand congruence in terms of rigid motions***
• **G-CO Building a tile pattern by reflecting hexagons** • **G-CO Building a tile pattern by reflecting octagons**	**6.** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the **definition** of congruence in terms of rigid motions to decide if they are congruent.
• **G-CO Properties of Congruent Triangles**	**7.** Use the **definition** of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
• **G-CO SSS Congruence Criterion** • **G-CO When Does SSA Work to Determine Triangle Congruence?** • **G-CO Why Does ASA Work?** • **G-CO Why does SAS work?** • **G-CO Why does SSS work?**	**8. Explain** how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the **definition** of congruence in terms of rigid motions.
	G.CO.C ***Prove geometric theorems***
• **G-C, G-CO Tangent Lines and the Radius of a Circle** • **G-CO Congruent angles made by parallel lines and a transverse** • **G-CO Points equidistant from two points in the plane**	**9. Prove** theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
• **G-C, G-SRT Seven Circles I** • **G-CO Classifying Triangles** • **G-CO Congruent angles in isosceles triangles** • **G-CO Midpoints of Triangle Sides** • **G-CO Sum of angles in a triangle** • **G-SRT Finding the Area of an Equilateral Triangle**	**10. Prove** theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
• **G-CO, G-SRT Congruence of parallelograms** • **G-CO Is this a parallelogram?** • **G-CO Midpoints of the Sides of a Parallelogram** • **G-CO Parallelograms and Translations**	**11. Prove** theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.
Domain-Similarity, Right Triangles, and Trigonometry	
Illustrations	**Cluster/Standard**
	G.SRT.A ***Understand similarity in terms of similarity transformations***
	1. Verify experimentally the properties of dilations given by a center and a scale factor: **a.** A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. **b.** The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Illustrations	Cluster/Standard
• G-SRT Are They Similar? • G-SRT Congruent and Similar Triangles • G-SRT Similar Quadrilaterals • G-SRT Similar Triangles	**2.** Given two figures, use the **definition** of similarity in terms of similarity transformations to decide if they are similar; **explain** using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
• G-SRT Similar triangles	**3.** Use the properties of similarity transformations to **establish** the AA criterion for two triangles to be similar.
	G.SRT.B ***Prove theorems involving similarity***
• G-SRT Joining two midpoints of sides of a triangle • G-SRT Pythagorean Theorem	**4. Prove** theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
• 8.G, G-GPE, G-SRT, G-CO Is this a rectangle? • 8.G, G-SRT Points from Directions • G-CO, G-SRT Congruence of parallelograms • G-GPE, G-CO, G-SRT Unit Squares and Triangles • G-GPE, G-SRT Finding triangle coordinates • G-GPE, G-SRT Slope Criterion for Perpendicular Lines • G-SRT Bank Shot • G-SRT Extensions, Bisections and Dissections in a Rectangle • G-SRT Folding a square into thirds • G-SRT, G-MG How far is the horizon? • G-SRT Tangent Line to Two Circles	**5.** Use congruence and similarity criteria for triangles to solve problems and to **prove** relationships in geometric figures.
• G-C, G-SRT Seven Circles I • G-SRT Finding the Area of an Equilateral Triangle • G-SRT Mt. Whitney to Death Valley	**G.SRT.C** ***Define trigonometric ratios and solve problems involving right triangles***
• G-SRT Defining Trigonometric Ratios • G-SRT Tangent of Acute Angles	**6.** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to **definitions** of trigonometric ratios for acute angles.
• G-SRT Sine and Cosine of Complementary Angles • G-SRT Trigonometric Function Values	**7. Explain** and use the relationship between the sine and cosine of complementary angles.
	G.SRT.D ***Apply trigonometry to general triangles***
	9. (+) Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
	10. (+) **Prove** the Laws of Sines and Cosines and use them to solve problems.
Domain--Circles	
Illustrations	**Cluster/Standard**
• G-C Two Wheels and a Belt	**G.C.A** ***Understand and apply theorems about circles***
• G-C Similar circles	**1. Prove** that all circles are similar.
• G-C, G-CO Tangent Lines and the Radius of a Circle • G-C, G-SRT Neglecting the Curvature of the Earth • G-C Right triangles inscribed in circles I • G-C Right triangles inscribed in circles II	**2. Identify** and **describe** relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

Illustrations	**Cluster/Standard**
• **G-C Circumcenter of a triangle** • **G-C Circumscribed Triangles** • **G.C Inscribing a circle in a triangle I** • **G.C Inscribing a circle in a triangle II** • **G-C Inscribing a triangle in a circle** • **G-C Locating Warehouse** • **G-C Opposite Angles in a Cyclic Quadrilateral** • **G-C Placing a Fire Hydrant**	**3.** Construct the inscribed and circumscribed circles of a triangle, and **prove** properties of angles for a quadrilateral inscribed in a circle.
• **G-C, G-SRT Setting Up Sprinklers** • **G-C Orbiting Satellite** • **G-C Two Wheels and a Belt**	**G.C.B** ***Find arc lengths and areas of sectors of circle***
• **G-C Mutually Tangent Circles**	**5. Derive** using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and **define** the radian measure of the angle as the constant of proportionality; **derive** the formula for the area of a sector.
Domain--Expressing Geometric Properties with Equations	
Illustrations	**Standard**
• **G-C Mutually Tangent Circles**	**G.GPE.A** ***Translate between the geometric description and the equation for a conic section***
• **G-GPE Explaining the equation for a circle** • **G-GPE Slopes and Circles**	**1. Derive** the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
• **G-GPE Defining Parabolas Geometrically-**	**2. Derive** the equation of a parabola given a focus and directrix
	3. (+) **Derive** the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
• **8.G, G-GPE, G-SRT, G-CO Is this a rectangle?** • **G-GPE, F-TF Coordinates of Points on a Circle** • **G-GPE, N-RN, F-TF Coordinates of equilateral triangles**	**G.GPE.B** ***Use coordinates to prove simple geometric theorems algebraically***
• **G-GPE A Midpoint Miracle** • **G-GPE, G-CO, G-SRT Unit Squares and Triangles**	**4.** Use coordinates to **prove** simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1,* $\sqrt{3}$*) lies on the circle centered at the origin and containing the point (0, 2)*
• **G-GPE.5, G-C.2, A-CED.2 Triangles inscribed in a circle** • **G-GPE A Midpoint Miracle** • **G-GPE Equal Area Triangles on the Same Base I** • **G-GPE Equal Area Triangles on the Same Base II** • **G-GPE, G-CO, G-SRT Unit Squares and Triangles** • **G-GPE, G-SRT Slope Criterion for Perpendicular Lines** • **G-GPE Parallel Lines in the Coordinate Plane** • **G-GPE When are two lines perpendicular?**	**5. Prove** the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Mathematics Assessment Project
CLASSROOM CHALLENGES
A Formative Assessment Lesson

Proofs of the Pythagorean Theorem

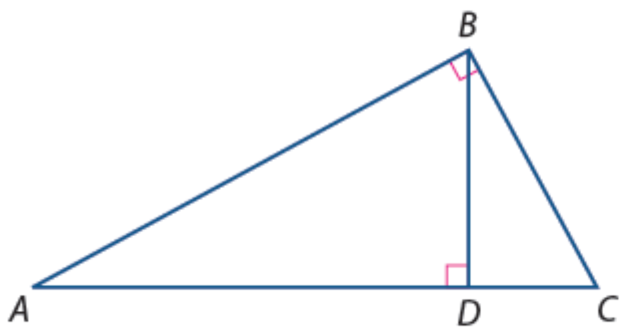


For example, he could write:

- The side length of the large square is $a+b$. So the area of the large square is $(a+b)^2 = a^2+b^2+2ab$. Now I can find the area of the individual pieces of the large square.
- The inner square has side length c and area c^2.
- Each of the right triangles has area $\frac{1}{2}ab$. Two of these triangles form a rectangle ab. There are four of them. So this gives an area of $2ab$ from the triangles.
- Adding together all the pieces gives the area of the large square. So the area of the large square is c^2+2ab.
- I now have two ways of writing the area of the large square. So $c^2+2ab= a^2+b^2+2ab$.
- Subtract $2ab$ from each side to see that $c^2= a^2+b^2$.

From Book 3 of Core-Plus, Unit 3 Looking Back

In the diagram below, $\triangle ABC$ is a right triangle and $\overline{BD}$ is an altitude to side $\overline{AC}$.



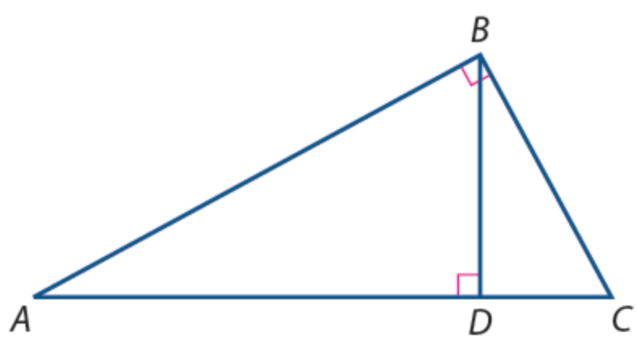
a. Prove that $(AB)^2 = (AC)(AD)$.

b. Find a similar expression for $(BC)^2$.

c. Use your work in Parts a and b to provide another proof of the Pythagorean Theorem.

Solution

a. **Prove:** $(AB)^2 = (AC)(AD)$



$\angle ADB \cong \angle ABC$ (right angles). $\angle A$ is common to both triangles. So, $\triangle ADB \sim \triangle ABC$ (AA). $\triangle ABC$ is related to $\triangle ADB$ by a scale factor $\frac{AB}{AD}$ which is the same value as $\frac{AC}{AB}$. So, $\frac{AB}{AD} = \frac{AC}{AB}$ and thus, $(AB)^2 = (AC)(AD)$.

b. $\triangle BDC \sim \triangle ABC$ (AA) by scale factor $\frac{BC}{DC}$.

$\frac{BC}{DC} = \frac{AC}{BC}$

So, $(BC)^2 = (AC)(DC)$.

c.
$$\begin{aligned}(AB)^2 + (BC)^2 &= (AC)(AD) + (AC)(DC)\\ &= (AC)(AD + DC)\\ &= (AC)(AC)\\ &= (AC)^2\end{aligned}$$

8.G Converse of the Pythagorean Theorem from Illustrative Mathematics

Task

A Pythagorean triple (a,b,c) is a set of three positive whole numbers which satisfy the equation

$$a^2 + b^2 = c^2$$

Many ancient cultures used simple Pythagorean triples such as (3,4,5) in order to accurately construct right angles: if a triangle has sides of lengths 3, 4, and 5 units, respectively, then the angle opposite the side of length 5 units is a right angle.

a. State the Pythagorean Theorem and its converse.

b. Explain why this practice of constructing a triangle with side-lengths 3, 4, and 5 to produce a right angle uses the converse of the Pythagorean Theorem.

c. Explain, in this particular case, why the converse of the Pythagorean Theorem is true.

Solution

a. The Pythagorean Theorem states that in a right triangle with side lengths a,b,c, with c being the length of the hypotenuse (that is, the side opposite the right angle), the relationship

$$a^2 + b^2 = c^2$$

always holds.

The converse of the Pythagorean Theorem says that *if* a,b,c are side lengths of a triangle that satisfy

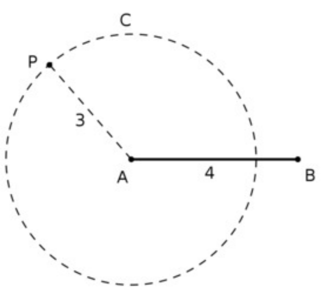
$$a^2 + b^2 = c^2$$

then the angle opposite the side of length c is a right angle.

b. The ancient cultures are trying to conclude that an angle is a right angle based on the side lengths of a triangle. Looking at part (a), it is the *converse* of the Pythagorean Theorem which has as its conclusion that an angle is a right angle so they are using the converse of the Pythagorean Theorem. Since 32+42=52, the converse of the Pythagorean Theorem implies that a triangle with side lengths 3,4,5 is a right triangle, the right angle being opposite the side of length 5.

To put this in other words, the Pythagorean Theorem tells us that a certain relation holds amongst the side lengths of a right triangle. The architects, however, do not have a right triangle but rather want to *produce* a right triangle. The converse of the Pythagorean Theorem enables them to do just this: they can conclude that an angle is a right angle provided a certain relationship holds between side lengths of a triangle.

c. Suppose AB is a line segment of length 4 units. The set of points in the plane whose distance from A is 3 units forms a circle C. If P is a point on C then the length |BP| could be as small as 1, if P is on segment AB, and as large as 7 if P is opposite B on line AB.



If we imagine the point P moving around the circle in either direction, the length of BP increases from 1 to 7 as P moves away from B. On the way, there are exactly two places where ∠PAB will be a right angle, namely when P is the northernmost point of C or when P is the southernmost point of C. In both cases, we can use the Pythagorean Theorem to compute the length of |BP| and find that it is 5 units. Note that if P is on the right side of the circle, its length will be less than when it is exactly vertical, and if it is on the left side of the circle, its length will be greater than when it is exactly vertical. So there are only two triangles that we can construct with side lengths 3, 4, and 5, and they happen only when the angle opposite the side with length 5 is a right angle. So if a triangle has side lengths 3, 4, and 5 units, it must be a right triangle.